2. $\frac{dy}{dx}=\frac{d}{dx}(\tan(e^x))$
$=\sec^2(e^x)\frac{d}{dx}(e^x)$
$=e^x\sec^2(e^x)$

3. $\frac{dy}{dx}=\frac{d}{dx}(\sin^3 x)$
$=\frac{d}{dx}((\sin x)^3)$
$=3(\sin x)^2\frac{d}{dx}(\sin x)$
$=3\sin^2 x\cos x$

4. $\frac{dy}{dx}=\frac{d}{dx}(\ln(\csc x))$
$=\frac{1}{\csc x}\frac{d}{dx}(\csc x)$
$=\frac{1}{\csc x}(-\csc x\cot x)$
$=-\cot x$

5. $\frac{ds}{dt}=\frac{d}{dt}\cos(1-2t)$
$=-\sin(1-2t)(-2)$
$=2\sin(1-2t)$

6. $\frac{ds}{dt}=\frac{d}{dt}\cot\left(\frac{2}{t}\right)$
$=-\csc^2\left(\frac{2}{t}\right)\frac{d}{dt}\left(\frac{2}{t}\right)$
$=-\csc^2\left(\frac{2}{t}\right)\left(-\frac{2}{t^2}\right)$
$=\frac{2}{t^2}\csc^2\left(\frac{2}{t}\right)$

7. $\frac{dy}{dx}=\frac{d}{dx}(\sqrt{1+\cos x})$
$=\frac{d}{dx}((1+\cos x)^{1/2})$
$=\frac{1}{2}(1+\cos x)^{-1/2}\frac{d}{dx}(1+\cos x)$
$=-\frac{\sin x}{2\sqrt{1+\cos x}}$

8. $\frac{dy}{dx}=\frac{d}{dx}(x\sqrt{2x+1})$
$=(x)\left(\frac{1}{2\sqrt{2x+1}}\right)(2)+(\sqrt{2x+1})(1)$
$=\frac{x+(2x+1)}{\sqrt{2x+1}}=\frac{3x+1}{\sqrt{2x+1}}$

9. $\frac{dr}{d\theta}=\frac{d}{d\theta}\sec(1+3\theta)$
$=\sec(1+3\theta)\tan(1+3\theta)(3)$
$=3\sec(1+3\theta)\tan(1+3\theta)$

10. $\frac{dr}{d\theta}=\frac{d}{d\theta}\tan^2(3-\theta^2)$
$=2\tan(3-\theta^2)\frac{d}{d\theta}\tan(3-\theta^2)$
$=2\tan(3-\theta^2)\sec^2(3-\theta^2)(-2\theta)$
$=-4\theta\tan(3-\theta^2)\sec^2(3-\theta^2)$

11. $\frac{dy}{dx}=\frac{d}{dx}\left(x^2\csc(5x)\right)$
$=(x^2)(-\csc(5x)\cot(5x))(5)$
$+(\csc(5x))(2x)$
$=-5x^2\csc(5x)\cot(5x)+2x\csc(5x)$

12. $\frac{dy}{dx}=\frac{d}{dx}\ln\sqrt{x}=\frac{1}{\sqrt{x}}\frac{d}{dx}\sqrt{x}$
$=\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2x},\ x>0$

13. $\frac{dy}{dx}=\frac{d}{dx}\ln(1+e^x)=\frac{1}{1+e^x}\frac{d}{dx}(1+e^x)=\frac{e^x}{1+e^x}$

14. $\frac{dy}{dx}=\frac{d}{dx}(xe^{-x})$
$=(x)(e^{-x})(-1)+(e^{-x})(1)$
$=-xe^{-x}+e^{-x}$

15. $\frac{dy}{dx}=\frac{d}{dx}(e^{1+\ln x})=\frac{d}{dx}(e^1e^{\ln x})=\frac{d}{dx}(ex)=e$

16. $\frac{dy}{dx}=\frac{d}{dx}\ln(\sin x)=\frac{1}{\sin x}\frac{d}{dx}(\sin x)$
$=\frac{\cos x}{\sin x}=\cot x,$
for values of x in the intervals $(k\pi,\ (k+1)\pi)$, where k is even.

17. $\frac{dr}{dx}=\frac{d}{dx}\ln(\cos^{-1}x)$
$=\frac{1}{\cos^{-1}x}\frac{d}{dx}\cos^{-1}x$
$=\frac{1}{\cos^{-1}x}\left(-\frac{1}{\sqrt{1-x^2}}\right)$
$=-\frac{1}{\cos^{-1}x\sqrt{1-x^2}}$ for $-1<x<1$

18. $\frac{dr}{d\theta} = \frac{d}{d\theta}\log_2(\theta^2)$
$= \frac{1}{\theta^2 \ln 2}\frac{d}{d\theta}(\theta^2)$
$= \frac{2\theta}{\theta^2 \ln 2}$
$= \frac{2}{\theta \ln 2}$

19. $\frac{ds}{dt} = \frac{d}{dt}\log_5(t-7) = \frac{1}{(t-7)\ln 5}\frac{d}{dt}(t-7)$
$= \frac{1}{(t-7)\ln 5}, t > 7$

20. $\frac{ds}{dt} = \frac{d}{dt}(8^{-t}) = 8^{-t}(\ln 8)\frac{d}{dt}(-t) = -8^{-t}\ln 8$

21. Use logarithmic differentiation.

$y = x^{\ln x}$
$\ln y = \ln(x^{\ln x})$
$\ln y = (\ln x)(\ln x)$
$\frac{d}{dx}\ln y = \frac{d}{dx}(\ln x)^2$
$\frac{1}{y}\frac{dy}{dx} = 2\ln x \frac{d}{dx}\ln x$
$\frac{dy}{dx} = \frac{2y\ln x}{x} = \frac{2x^{\ln x}\ln x}{x}$

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22. $\frac{dy}{dx} = \frac{d}{dx}\frac{(2x)2^x}{\sqrt{x^2+1}}$
$= \frac{\sqrt{x^2+1}\frac{d}{dx}[(2x)2^x] - (2x)(2^x)\frac{d}{dx}\sqrt{x^2+1}}{x^2+1}$
$= \frac{\sqrt{x^2+1}\left[(2x)(2^x)(\ln 2) + (2^x)(2)\right] - (2x)(2^x)\frac{1}{2\sqrt{x^2+1}}(2x)}{x^2+1}$
$= \frac{(x^2+1)(2^x)(2x\ln 2 + 2) - 2x^2(2^x)}{(x^2+1)^{3/2}}$
$= \frac{(2\cdot 2^x)[(x^2+1)(x\ln 2+1) - x^2]}{(x^2+1)^{3/2}}$
$= \frac{(2\cdot 2^x)(x^3\ln 2 + x^2 + x\ln 2 + 1 - x^2)}{(x^2+1)^{3/2}}$
$= \frac{(2\cdot 2^x)(x^3\ln 2 + x\ln 2 + 1)}{(x^2+1)^{3/2}}$

Alternate solution, using logarithmic differentiation:

$y = \frac{(2x)2^x}{\sqrt{x^2+1}}$
$\ln y = \ln(2x) + \ln(2^x) - \ln\sqrt{x^2+1}$
$\ln y = \ln 2 + \ln x + x\ln 2 - \frac{1}{2}\ln(x^2+1)$
$\frac{d}{dx}\ln y = \frac{d}{dx}[\ln 2 + \ln x + x\ln 2 - \frac{1}{2}\ln(x^2+1)]$
$\frac{1}{y}\frac{dy}{dx} = 0 + \frac{1}{x} + \ln 2 - \frac{1}{2}\frac{1}{x^2+1}(2x)$
$\frac{dy}{dx} = y\left(\frac{1}{x} + \ln 2 - \frac{x}{x^2+1}\right) = \frac{(2x)2^x}{\sqrt{x^2+1}}\left(\frac{1}{x} + \ln 2 - \frac{x}{x^2+1}\right)$

23. $\frac{dy}{dx} = \frac{d}{dx}e^{\tan^{-1}x} = e^{\tan^{-1}x}\frac{d}{dx}\tan^{-1}x = \frac{e^{\tan^{-1}x}}{1+x^2}$

24. $\frac{dy}{du}=\frac{d}{du}\sin^{-1}\sqrt{1-u^2}=\frac{1}{\sqrt{1-\left(\sqrt{1-u^2}\right)^2}}\frac{d}{du}\sqrt{1-u^2}=\frac{1}{\sqrt{u^2}}\frac{1}{2\sqrt{1-u^2}}(-2u)=\frac{-u}{|u|\sqrt{1-u^2}}$

25. $\frac{dy}{dt}=\frac{d}{dt}\left(t\sec^{-1}t-\frac{1}{2}\ln t\right)=(t)\left(\frac{1}{|t|\sqrt{t^2-1}}\right)+(\sec^{-1}t)(1)-\frac{1}{2t}=\frac{t}{|t|\sqrt{t^2-1}}+\sec^{-1}t-\frac{1}{2t}$

26. $\frac{dy}{dt}=\frac{d}{dt}\left[(1+t^2)\cot^{-1}(2t)\right]=(1+t^2)\left(-\frac{1}{1+(2t)^2}\right)(2)+(\cot^{-1}(2t))\,(2t)=-\frac{2+2t^2}{1+4t^2}+2t\cot^{-1}(2t)$

27. $\frac{dy}{dz}=\frac{d}{dz}\left(z\cos^{-1}z-\sqrt{1-z^2}\right)$

$=(z)\left(-\frac{1}{\sqrt{1-z^2}}\right)+(\cos^{-1}z)(1)-\frac{1}{2\sqrt{1-z^2}}(-2z)$

$=-\frac{z}{\sqrt{1-z^2}}+\cos^{-1}z+\frac{z}{\sqrt{1-z^2}}=\cos^{-1}z$

28. $\frac{dy}{dx}=\frac{d}{dx}(2\sqrt{x-1}\csc^{-1}\sqrt{x})$

$=\left(2\sqrt{x-1}\right)\left(-\frac{1}{|\sqrt{x}|\sqrt{(\sqrt{x})^2-1}}\right)\left(\frac{1}{2\sqrt{x}}\right)+\left(2\csc^{-1}\sqrt{x}\right)\left(\frac{1}{2\sqrt{x-1}}\right)$

$=-\frac{\sqrt{x-1}}{(\sqrt{x})^2\sqrt{x-1}}+\frac{\csc^{-1}\sqrt{x}}{\sqrt{x-1}}=-\frac{1}{x}+\frac{\csc^{-1}\sqrt{x}}{\sqrt{x-1}}$

29. $\frac{dy}{dx}=\frac{d}{dx}\csc^{-1}(\sec x)=\left(-\frac{1}{|\sec x|\sqrt{\sec^2 x-1}}\right)\frac{d}{dx}(\sec x)$

$=-\frac{1}{|\sec x|\sqrt{\tan^2 x}}\sec x\tan x=-\frac{\sec x\tan x}{|\sec x\tan x|}$

$=-\frac{\frac{1}{\cos x}\frac{\sin x}{\cos x}}{\left|\frac{1}{\cos x}\frac{\sin x}{\cos x}\right|}=-\frac{\sin x}{|\sin x|}=-1 \text{ for } 0\le x<\frac{\pi}{2}$

Alternate method:

On the domain $0\le x<\frac{\pi}{2}$, we may rewrite the function as follows:

$y=\csc^{-1}(\sec x)=\frac{\pi}{2}-\sec^{-1}(\sec x)=\frac{\pi}{2}-\cos^{-1}(\cos x)=\frac{\pi}{2}-x$

Therefore, $\frac{dy}{dx}=-1$ for $0\le x<\frac{\pi}{2}$.

Note that the derivative exists at 0 only because this is an endpoint of the given domain; the two-sided derivative of $y=\csc^{-1}(\sec x)$ does not exist at this point.

30. $$\frac{dr}{d\theta}=\frac{d}{d\theta}\left(\frac{1+\sin\theta}{1-\cos\theta}\right)^2$$
$$=2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{(1-\cos\theta)(\cos\theta)-(1+\sin\theta)(\sin\theta)}{(1-\cos\theta)^2}\right)$$
$$=2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{\cos\theta-\cos^2\theta-\sin\theta-\sin^2\theta}{(1-\cos\theta)^2}\right)$$
$$=2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{\cos\theta-\sin\theta-1}{(1-\cos\theta)^2}\right)$$

31. Because $y=\ln\left(x^2\right)$ is defined for all $x\neq 0$ and $\frac{dy}{dx}=\frac{1}{x^2}\frac{d}{dx}(x^2)=\frac{2x}{x^2}=\frac{2}{x}$, the function is differentiable for all $x\neq 0$.

32. Because $y=\sin(e^{2x})$ is defined for all real x and $\frac{dy}{dx}=2e^{2x}\cos(e^{2x})$, the function is differentiable for all real x.

33. Because $y=\sqrt{\frac{1-x}{1+x^2}}$ is defined for all $x<1$ and
$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1-x}{1+x^2}\right)^{-1/2}\frac{(1+x^2)(-1)-(1-x)(2x)}{(1+x^2)^2}$$
$$=\frac{x^2-2x-1}{2\sqrt{1-x}(1+x^2)^{3/2}},$$
which is defined only for $x<1$, the function is differentiable for all $x<1$.

34. Because $y=\frac{1}{1-e^x}$ is defined for all $x\neq 0$ and
$$\frac{dy}{dx}=\frac{d}{dx}((1-e^x)^{-1})$$
$$=-(1-e^x)^{-2}(-e^x)=\frac{e^x}{(1-e^x)^2},$$
the function is differentiable for all $x\neq 0$.

35. Use implicit differentiation.
$$xy+2x+3y=1$$
$$\frac{d}{dx}(xy)+\frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}(1)$$
$$x\frac{dy}{dx}+y(1)+2+3\frac{dy}{dx}=0$$
$$(x+3)\frac{dy}{dx}=-(y+2)$$
$$\frac{dy}{dx}=-\frac{y+2}{x+3}$$

36. Use implicit differentiation.
$$5x^{4/5}+10y^{6/5}=15$$
$$\frac{d}{dx}\left(5x^{4/5}\right)+\frac{d}{dx}\left(10y^{6/5}\right)=\frac{d}{dx}(15)$$
$$4x^{-1/5}+12y^{1/5}\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{4x^{-1/5}}{12y^{1/5}}$$
$$=-\frac{1}{3(xy)^{1/5}}$$

37. Use implicit differentiation.
$$\sqrt{xy}=1$$
$$\frac{d}{dx}\sqrt{xy}=\frac{d}{dx}(1)$$
$$\frac{1}{2\sqrt{xy}}\left[x\frac{dy}{dx}+(y)(1)\right]=0$$
$$x\frac{dy}{dx}+y=0$$
$$\frac{dy}{dx}=-\frac{y}{x}$$
Alternate method:
$$\ln\sqrt{xy}=\ln(1)$$
$$\frac{1}{2}[\ln x+\ln y]=0$$
$$\ln x+\ln y=0$$
$$\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{y}{x}$$

38. Use implicit differentiation.

$$y^2 = \frac{x}{x+1}$$
$$\frac{d}{dx}y^2 = \frac{d}{dx}\frac{x}{x+1}$$
$$2y\frac{dy}{dx} = \frac{(x+1)(1)-(x)(1)}{(x+1)^2}$$
$$\frac{dy}{dx} = \frac{1}{2y(x+1)^2}$$

39.

$$x^3 + y^3 = 1$$
$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = \frac{d}{dx}(1)$$
$$3x^2 + 3y^2y' = 0$$
$$y' = -\frac{x^2}{y^2}$$

$$y'' = \frac{d}{dx}\left(-\frac{x^2}{y^2}\right)$$
$$= -\frac{(y^2)(2x)-(x^2)(2y)(y')}{y^4}$$
$$= -\frac{(y^2)(2x)-(x^2)(2y)\left(-\frac{x^2}{y^2}\right)}{y^4}$$
$$= -\frac{2xy^3+2x^4}{y^5} = -\frac{2x(x^3+y^3)}{y^5} = -\frac{2x}{y^5}$$

because $x^3 + y^3 = 1$.

40.

$$y^2 = 1 - \frac{2}{x}$$
$$\frac{d}{dx}(y^2) = \frac{d}{dx}(1) - \frac{d}{dx}\left(\frac{2}{x}\right)$$
$$2yy' = \frac{2}{x^2}$$
$$y' = \frac{2}{x^{2(2y)}} = \frac{1}{x^2y}$$
$$y'' = \frac{d}{dx}\left(\frac{1}{x^2y}\right) = -\frac{1}{(x^2y)^2}\frac{d}{dx}(x^2y)$$
$$= -\frac{1}{(x^2y)^2}[(x^2)(y') + (y)(2x)]$$
$$= -\frac{1}{(x^2y)^2}\left[(x^2)\left(\frac{1}{x^2y}\right) + 2xy\right]$$
$$= -\frac{1}{x^4y^2}\left(\frac{1}{y} + 2xy\right) = -\frac{1+2xy^2}{x^4y^3}$$

41.

$$y^3 + y = 2\cos x$$
$$\frac{d}{dx}(y^3) + \frac{d}{dx}(y) = \frac{d}{dx}(2\cos x)$$
$$3y^2y' + y' = -2\sin x$$
$$(3y^2+1)y' = -2\sin x$$
$$y' = -\frac{2\sin x}{3y^2+1}$$
$$y'' = \frac{d}{dx}\left(-\frac{2\sin x}{3y^2+1}\right)$$
$$= -\frac{(3y^2+1)(2\cos x) - (2\sin x)(6yy')}{(3y^2+1)^2}$$
$$= -\frac{(3y^2+1)(2\cos x) - (12y\sin x)\left(-\frac{2\sin x}{3y^2+1}\right)}{(3y^2+1)^2}$$
$$= -2\frac{(3y^2+1)^2\cos x + 12y\sin^2 x}{(3y^2+1)^3}$$

42.

$$x^{1/3} + y^{1/3} = 4$$
$$\frac{d}{dx}(x^{1/3}) + \frac{d}{dx}(y^{1/3}) = \frac{d}{dx}(4)$$
$$\frac{1}{3}x^{-2/3} + \frac{1}{3}y^{-2/3}y' = 0$$
$$y' = -\frac{x^{-2/3}}{y^{-2/3}} = -\left(\frac{y}{x}\right)^{2/3}$$
$$y'' = \frac{d}{dx}\left[-\left(\frac{y}{x}\right)^{2/3}\right]$$
$$= -\frac{2}{3}\left(\frac{y}{x}\right)^{-1/3}\left(\frac{xy'-(y)(1)}{x^2}\right)$$
$$= -\frac{2}{3}\left(\frac{y}{x}\right)^{-1/3}\left(\frac{(x)\left[-\left(\frac{y}{x}\right)^{2/3}\right] - y}{x^2}\right)$$
$$= -\frac{2}{3}x^{1/3}y^{-1/3}(-x^{-5/3}y^{2/3} - x^{-2}y)$$
$$= \frac{2}{3}x^{-4/3}y^{1/3} + \frac{2}{3}x^{-5/3}y^{2/3}$$

43. $\dfrac{d^{40}y}{dx^{40}} = e^{x\sqrt[8]{2}}\left(\sqrt[8]{2}\right)^{40} = 2^5 e^{x\sqrt[8]{2}} = 32e^{x\sqrt[8]{2}}$

44. Note that the 4th, 8th, ... and 40th derivatives of $\sin x$ cycle back to $\sin x$. By the chain rule, each derivative generates another factor of $\sqrt[8]{2}$. Thus

$$\frac{d^{40}y}{dx^{40}} = \sin\left(x\sqrt[8]{2}\right)\left(\sqrt[8]{2}\right)^{40} = 32\sin\left(x\sqrt[8]{2}\right).$$

45. $\frac{dy}{dx} = \frac{d}{dx}\sqrt{x^2 - 2x} = \frac{1}{2\sqrt{x^2 - 2x}}(2x - 2) = \frac{x - 1}{\sqrt{x^2 - 2x}}$

At $x = 3$, $y = \sqrt{3^2 - 2(3)} = \sqrt{3}$ and $\frac{dy}{dx} = \frac{3 - 1}{\sqrt{3^2 - 2(3)}} = \frac{2}{\sqrt{3}}$.

(a) Tangent: $y = \sqrt{3} + \frac{2}{\sqrt{3}}(x - 3)$

(b) Normal: $y = \sqrt{3} - \frac{\sqrt{3}}{2}(x - 3)$

46. $\frac{dy}{dx} = \frac{d}{dx}(\tan(2x)) = 2\sec^2(2x)$

At $x = \frac{\pi}{3}$, $y = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$ and $\frac{dy}{dx} = 2\sec^2\frac{2\pi}{3} = 2(-2)^2 = 8$.

(a) Tangent: $y = -\sqrt{3} + 8\left(x - \frac{\pi}{3}\right)$

(b) Normal: $y = -\sqrt{3} - \frac{1}{8}\left(x - \frac{\pi}{3}\right)$

47. Use implicit differentiation.

$$x^2 + 2y^2 = 9$$
$$\frac{d}{dx}(x^2) + \frac{d}{dx}(2y^2) = \frac{d}{dx}(9)$$
$$2x + 4y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{2x}{4y} = -\frac{x}{2y}$$

Slope at (1, 2): $-\frac{1}{2(2)} = -\frac{1}{4}$

(a) Tangent: $y = 2 - \frac{1}{4}(x - 1)$

(b) Normal: $y = 2 + 4(x - 1)$

48. Use implicit differentiation.

$$x + \sqrt{xy} = 6$$
$$\frac{d}{dx}(x) + \frac{d}{dx}\left(\sqrt{xy}\right) = \frac{d}{dx}(6)$$
$$1 + \frac{1}{2\sqrt{xy}}\left[(x)\left(\frac{dy}{dx}\right) + (y)(1)\right] = 0$$
$$\frac{x}{2\sqrt{xy}}\frac{dy}{dx} = -1 - \frac{y}{2\sqrt{xy}}$$
$$\frac{dy}{dx} = \frac{2\sqrt{xy}}{x}\left(-1 - \frac{y}{2\sqrt{xy}}\right)$$
$$= -2\sqrt{\frac{y}{x}} - \frac{y}{x}$$

Slope at (4, 1): $-2\sqrt{\frac{1}{4}} - \frac{1}{4} = -\frac{2}{2} - \frac{1}{4} = -\frac{5}{4}$

(a) Tangent: $y - 1 = -\frac{5}{4}(x - 4)$

(b) Normal: $y - 1 = \frac{4}{5}(x - 4)$

49. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{2\cos t} = -\tan t$

At $t = \frac{3\pi}{4}$, we have $x = 2\sin\frac{3\pi}{4} = \sqrt{2}$,

$y = 2\cos\frac{3\pi}{4} = -\sqrt{2}$, and $\frac{dy}{dx} = -\tan\frac{3\pi}{4} = 1$.

The equation of the tangent line is

$y = -\sqrt{2} + 1\left(x - \sqrt{2}\right)$.

50. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4\cos t}{-3\sin t} = -\frac{4}{3}\cot t$

At $t = \frac{3\pi}{4}$, we have $x = 3\cos\frac{3\pi}{4} = -\frac{3\sqrt{2}}{2}$,

$y = 4\sin\frac{3\pi}{4} = 2\sqrt{2}$, and $\frac{dy}{dx} = -\frac{4}{3}\cot\frac{3\pi}{4} = \frac{4}{3}$.

The equation of the tangent line is

$y = 2\sqrt{2} + \frac{4}{3}\left(x + \frac{3\sqrt{2}}{2}\right)$.

51. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{5\sec^2 t}{3\sec t\tan t} = \frac{5\sec t}{3\tan t} = \frac{5}{3\sin t}$

At $t = \frac{\pi}{6}$, we have $x = 3\sec\left(\frac{\pi}{6}\right) = 2\sqrt{3}$,

$y = 5\tan\left(\frac{\pi}{6}\right) = \frac{5\sqrt{3}}{3}$, and $\frac{dy}{dx} = \frac{5}{3\sin\left(\frac{\pi}{6}\right)} = \frac{10}{3}$.

The equation of the tangent line is

$y = \frac{5\sqrt{3}}{3} + \frac{10}{3}\left(x - 2\sqrt{3}\right)$.

52. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1+\cos t}{-\sin t}$

At $t = -\frac{\pi}{4}$, we have $x = \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$,

$y = -\frac{\pi}{4} + \sin\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4} - \frac{\sqrt{2}}{2}$, and

$\frac{dy}{dx} = \frac{1+\cos\left(-\frac{\pi}{4}\right)}{-\sin\left(-\frac{\pi}{4}\right)} = \frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{2} + 1$.

The equation of the tangent line is

$y = -\left(\frac{\pi}{4} + \frac{\sqrt{2}}{2}\right) + \left(1 + \sqrt{2}\right)\left(x - \frac{\sqrt{2}}{2}\right)$.

53. **(a)** $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (\sin ax + b\cos x) = b$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (5x+3) = 3$. Thus $\lim_{x\to 0} f(x) = f(0) = 3$ if and only if $b = 3$.

(b) $f'(x) = \begin{cases} a\cos ax - b\sin x, & x < 0 \\ 5, & x > 0 \end{cases}$

The slopes match at $x = 0$ if and only if $a = 5$.

(c) No; although the slopes match, the function is not continuous.

54. **(a)** The function is continuous for all values of m, because the right-hand limit as $x \to 0$ is equal to $f(0) = 0$ for any value of m.

(b) The left-hand derivative at $x = 0$ is $2\cos(2 \cdot 0) = 2$, and the right-hand derivative at $x = 0$ is m, so in order for the function to be differentiable at $x = 0$, m must be 2.

55. Note that $f(x) = \sqrt[7]{(x-1)^3} = (x-1)^{3/7}$, so $f'(x) = \frac{3}{7}(x-1)^{-4/7}$, which is defined if and only if $x \neq 1$. Thus the answers are

(a) For all $x \neq 1$ **(b)** At $x = 1$

(c) Nowhere

56. **(a)** For all x **(b)** Nowhere

(c) Nowhere

57. The individual functions are continuous and differentiable on $[-1, 3]$. At $x = 1$ we have $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{x^2+3} = 2$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x+1) = 2 = f(1)$, so f is continuous at $x = 1$. However,

$f'(x) = \begin{cases} \frac{2x}{2\sqrt{x^2+3}}, & x < 1 \\ 1, & x > 1 \end{cases}$, so the slope is $\frac{1}{2}$

coming from the left and 1 coming from the right. Thus f is not differentiable at $x = 1$. The answers are:

(a) $[-1, 1) \cup (1, 3]$ **(b)** At $x = 1$

(c) Nowhere

58. The individual functions are continuous and differentiable on [–3, 3]. At $x = 0$ we have $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sin 2x = 0$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x^2+2x+1) = 1 = f(1)$, so f is not continuous at $x = 0$. Thus f is not differentiable at $x = 0$. The answers are:

(a) $[-3, 0)\cup(0, 3]$

(b) Nowhere

(c) At $x = 0$

59. (a) Because $3 - \sin x > 0$ for all x,

$$y = \left(\sqrt{(3-\sin x)}\right)^2 = 3 - \sin x, \text{ so}$$

$$\frac{dy}{dx} = -\cos x.$$

(b)
$$y = \ln(3e^{7x^2-13x+5}) = \ln 3 + \ln(e^{7x^2-13x+5}) = \ln 3 + 7x^2 - 13x + 5,$$

so $\frac{dy}{dx} = 14x - 13.$

(c) $s = \tan(\tan^{-1}(t^2-3t)) = t^2 - 3t$, so

$$\frac{ds}{dt} = 2t - 3.$$

(d) $s = \sqrt[3]{t^6} - 5(\sin(\sin^{-1} t))^6 = t^2 - 5t^6$, so

$$\frac{ds}{dt} = 2t - 30t^5.$$

60. (a) $y = \ln\left(\frac{2x+7}{3x+2}\right) = \ln(2x+7) - \ln(3x+2)$,

so $\frac{dy}{dx} = \frac{2}{2x+7} - \frac{3}{3x+2}.$

(b)
$$y = \frac{(x^2-1)^2}{(x^2-2x+1)(x+1)} = \frac{(x-1)^2(x+1)^2}{(x-1)^2(x+1)} = x+1,$$

so $\frac{dy}{dx} = 1.$

(c) $s = \sin^2(\cos^{-1} t) = 1 - \cos^2(\cos^{-1} t) = 1 - t^2$,

so $\frac{ds}{dt} = -2t.$

(d)
$$s = \left(\frac{2\sqrt{t}}{\sqrt[3]{t}}\right)^5 = \left(2\cdot t^{\frac{1}{2}-\frac{1}{3}}\right)^5 = (2t^{1/6})^5 = 32t^{5/6},$$

so $\frac{ds}{dt} = \left(\frac{160}{6}\right)t^{-1/6} = \frac{80}{3\sqrt[6]{t}}.$

61. (a) The line passes through (2, 4), which must be the point of tangency. The slope of the tangent line is 3, so the slope of the normal line is $-\frac{1}{3}$. An equation of the normal line is $y = 4 - \frac{1}{3}(x-2)$.

(b) The tangent line to f at (2, 4) has slope 3, so the tangent line to f^{-1} at (4, 2) has slope $\frac{1}{3}$. An equation of the line is

$$y = 2 + \frac{1}{3}(x-4).$$

(c) When $x = 2$, $y = \frac{f(2)}{2} = \frac{4}{2} = 2$ and

$$\frac{dy}{dx} = \left.\frac{f'(x)\cdot x - 1\cdot f(x)}{x^2}\right|_{x=2} = \frac{3\cdot 2 - 4}{4} = \frac{1}{2}.$$

An equation of the line is $y = 2 + \frac{1}{2}(x-2)$.

62. (a) The line passes through (1, 3), which must be the point of tangency. The slope of the tangent line is –2, so the slope of the normal line is $\frac{1}{2}$. An equation of the normal line is $y = 3 + \frac{1}{2}(x-1)$.

(b) The tangent line to g at (1, 3) has slope –2, so the tangent line to g^{-1} at (3, 1) has slope $-\frac{1}{2}$. An equation of the line is

$$y = 1 - \frac{1}{2}(x-3).$$

(c) When $x = 1$, $y = g(1^2) = g(1) = 3$ and $\frac{dy}{dx} = g'(x^2)\cdot 2x\big|_{x=1} = g'(1)\cdot 2 = -4$. An equation of the line is $y = 3 - 4(x-1)$.

63. First, note that
$\ln y = 5\ln(x+2) + 4\ln(2x-3) - 2\ln(x+17)$.
Then $\frac{1}{y}\frac{dy}{dx} = \frac{5}{x+2} + \frac{8}{2x-3} - \frac{2}{x+17}$, and so

$$\frac{dy}{dx} = \frac{(x+2)^5(2x-3)^4}{(x+17)^2}\left(\frac{5}{x+2} + \frac{8}{2x-3} - \frac{2}{x+17}\right)$$

64. First, note that $\ln y = (x+5)\ln(x^2+2)$. Then
$\frac{1}{y}\frac{dy}{dx} = 1\cdot\ln(x^2+2) + (x+5)\left(\frac{2x}{x^2+2}\right)$, and so

$$\frac{dy}{dx} = (x^2+2)^{x+5}\left(\ln(x^2+2) + \frac{2x^2+10x}{x^2+2}\right).$$

65. (a) $f(x) = \frac{x^2}{2}$ or $f(x) = \frac{x^2}{2} + C$

(b) $f(x) = e^x$ or $f(x) = Ce^x$

(c) $f(x) = e^{-x}$ or $f(x) = Ce^{-x}$

(d) $f(x) = e^x$ or $f(x) = e^{-x}$ or $f(x) = Ce^x + De^{-x}$

(e) $f(x) = \sin x$ or $f(x) = \cos x$ or $f(x) = C\sin x + D\cos x$

66. (a) $\frac{d}{dx}\left[\sqrt{x}f(x)\right] = \sqrt{x}f'(x) + \frac{1}{2\sqrt{x}}f(x)$

At $x = 1$, the derivative is

$$\sqrt{1}f'(1) + \frac{1}{2\sqrt{1}}f(1) = 1\left(\frac{1}{5}\right) + \left(\frac{1}{2}\right)(-3) = -\frac{13}{10}.$$

(b) $\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}}f'(x) = \frac{f'(x)}{2\sqrt{f(x)}}$

At $x = 0$, the derivative is

$$\frac{f'(0)}{2\sqrt{f(0)}} = -\frac{2}{2\sqrt{9}} = -\frac{1}{3}.$$

(c) $\frac{d}{dx}f(\sqrt{x}) = f'(\sqrt{x})\frac{d}{dx}\sqrt{x} = \frac{f'(\sqrt{x})}{2\sqrt{x}}$

At $x = 1$, the derivative is

$$\frac{f'(\sqrt{1})}{2\sqrt{1}} = \frac{f'(1)}{2} = \frac{\frac{1}{5}}{2} = \frac{1}{10}.$$

(d) $\frac{d}{dx}f(1 - 5\tan x) = f'(1 - 5\tan x)(-5\sec^2 x)$

At $x = 0$, the derivative is

$$f'(1 - 5\tan 0)(-5\sec^2 0) = f'(1)(-5) = \left(\frac{1}{5}\right)(-5) = -1.$$

(e) $\frac{d}{dx}\frac{f(x)}{2+\cos x} = \frac{(2+\cos x)(f'(x)) - (f(x))(-\sin x)}{(2+\cos x)^2}$

At $x = 0$, the derivative is

$$\frac{(2+\cos 0)(f'(0)) - (f(0))(-\sin 0)}{(2+\cos 0)^2} = \frac{3f'(0)}{3^2} = -\frac{2}{3}.$$

(f) $\frac{d}{dx}\left[10\sin\left(\frac{\pi x}{2}\right)f^2(x)\right]$

$$= 10\left(\sin\frac{\pi x}{2}\right)(2f(x)f'(x)) + 10f^2(x)\left(\cos\frac{\pi x}{2}\right)\left(\frac{\pi}{2}\right)$$

$$= 20f(x)f'(x)\sin\frac{\pi x}{2} + 5\pi f^2(x)\cos\frac{\pi x}{2}$$

At $x = 1$, the derivative is

$$20f(1)f'(1)\sin\frac{\pi}{2} + 5\pi f^2(1)\cos\frac{\pi}{2} = 20(-3)\left(\frac{1}{5}\right)(1) + 5\pi(-3)^2(0) = -12.$$

67. (a) $\frac{d}{dx}\left(\frac{f(2x)}{x-1}\right) = \frac{f'(2x)\cdot 2\cdot(x-1) - 1\cdot f(2x)}{(x-1)^2}$

At $x = 0$, the derivative is

$$\frac{f'(0)(-2) - f(0)}{1} = \frac{(-2)(-2) - (-1)}{1} = 5.$$

(b) $\frac{d}{dx}\left[f^2(x)g^3(x)\right]$
$= f^2(x)\cdot 3g^2(x)g'(x) + g^3(x)\cdot 2f(x)f'(x)$
$= f(x)g^2(x)\left[3f(x)g'(x) + 2g(x)f'(x)\right]$
At $x = 0$, the derivative is
$f(0)g^2(0)\left[3f(0)g'(0) + 2g(0)f'(0)\right]$
$= (-1)(-3)^2\left[3(-1)(4) + 2(-3)(-2)\right]$
$= -9\left[-12 + 12\right] = 0.$

(c) $\frac{d}{dx}g(f(x)) = g'(f(x))f'(x)$
At $x = -1$, the derivative is
$g'(f(-1))f'(-1) = g'(0)f'(-1)$
$= (4)(2) = 8.$

(d) $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$
At $x = -1$, the derivative is
$f'(g(-1))g'(-1) = f'(-1)g'(-1)$
$= (2)(1) = 2.$

(e) $\frac{d}{dx}f(g(2x-1)) = 2f'(g(2x-1))g'(2x-1)$
At $x = 0$, the derivative is
$2f'(g(-1))g'(-1) = 2f'(-1)g'(-1)$
$= 2(2)(1) = 4.$

(f) $\frac{d}{dx}g(x + f(x))$
$= g'(x + f(x))\frac{d}{dx}(x + f(x))$
$= g'(x + f(x))(1 + f'(x))$
At $x = 0$, the derivative is
$g'(0 + f(0))[1 + f'(0)] = g'(0 - 1)[1 + (-2)]$
$= (1)(-1) = -1$

68. $\frac{dw}{ds} = \frac{dw}{dr}\frac{dr}{ds}$

$$= \frac{d}{dr}\left[\sin\left(\sqrt{r} - 2\right)\right]\frac{d}{ds}\left[8\sin\left(s + \frac{\pi}{6}\right)\right]$$

$$= \left[\cos\left(\sqrt{r} - 2\right)\frac{1}{2\sqrt{r}}\right]\left[8\cos\left(s + \frac{\pi}{6}\right)\right]$$

At $s = 0$, we have $r = 8\sin\left(0 + \frac{\pi}{6}\right) = 4$ and so

$$\frac{dw}{ds} = \left[\cos\left(\sqrt{4} - 2\right)\frac{1}{2\sqrt{4}}\right]\left[8\cos\left(0 + \frac{\pi}{6}\right)\right]$$

$$= \left(\frac{\cos 0}{4}\right)\left(8\cos\frac{\pi}{6}\right) = \left(\frac{1}{4}\right)\left(\frac{8\sqrt{3}}{2}\right) = \sqrt{3}$$

69. Solving $\theta^2 t + \theta = 1$ for t, we have
$t = \frac{1-\theta}{\theta^2} = \theta^{-2} - \theta^{-1}$, and we may write:

$$\frac{dr}{d\theta} = \frac{dr}{dt}\frac{dt}{d\theta}$$

$$\frac{d}{d\theta}(\theta^2 + 7)^{1/3} = \frac{dr}{dt}\frac{d}{d\theta}(\theta^{-2} - \theta^{-1})$$

$$\frac{1}{3}(\theta^2 + 7)^{-2/3}(2\theta) = \left(\frac{dr}{dt}\right)(-2\theta^{-3} + \theta^{-2})$$

$$\frac{dr}{dt} = \frac{2\theta(\theta^2 + 7)^{-2/3}}{3(-2\theta^{-3} + \theta^{-2})} = \frac{2\theta^4(\theta^2 + 7)^{-2/3}}{3(\theta - 2)}$$

At $t = 0$, we may solve $\theta^2 t + \theta = 1$ to obtain $\theta = 1$, and so

$$\frac{dr}{dt} = \frac{2(1)^4(1^2 + 7)^{-2/3}}{3(1-2)} = \frac{2(8)^{-2/3}}{-3} = -\frac{1}{6}.$$

70. **(a)** One possible answer:
$x(t) = 10\cos\left(t + \frac{\pi}{4}\right),\ y(t) = 0$

(b) $s(0) = 10\cos\left(\frac{\pi}{4}\right) = 5\sqrt{2}$

(c) Farthest left:
When $\cos\left(t + \frac{\pi}{4}\right) = -1$, we have
$s(t) = -10.$
Farthest right:
When $\cos\left(t + \frac{\pi}{4}\right) = 1$, we have $s(t) = 10.$

(d) Because $\cos\frac{\pi}{2} = 0$, the particle first reaches the origin at $t = \frac{\pi}{4}$. The velocity is given by $v(t) = -10\sin\left(t + \frac{\pi}{4}\right)$, so the velocity at $t = \frac{\pi}{4}$ is $-10\sin\left(\frac{\pi}{2}\right) = -10$, and the speed at $t = \frac{\pi}{4}$ *is* $|-10| = 10$. The acceleration is given by
$a(t) = -10\cos\left(t + \frac{\pi}{4}\right)$, so the acceleration at $t = \frac{\pi}{4}$ is $-10\cos\left(\frac{\pi}{2}\right) = 0.$

71. **(a)** $8x+8(xy'+y)+2yy'=0$, so $y'=-\dfrac{4x+4y}{4x+y}$. Tangent lines are horizontal where $y'=0$, which is where $4x+4y=0$, in which case $y=-x$. Letting $y=-x$ in the equation of the hyperbola, we obtain $4x^2+8x(-x)+(-x)^2+3=0$. Solving yields $x=\pm 1$. Recalling that $y=-x$, we get the points $A(-1, 1)$; $B(1, -1)$.

(b) Again, $y'=-\dfrac{4x+4y}{4x+y}$. Tangent lines are vertical where y' is undefined, which is where $4x+y=0$, in which case $y=-4x$. Letting $y=-4x$ in the equation of the hyperbola, we obtain $4x^2+8x(-4x)+(-4x)^2+3=0$. Solving yields $x=\pm 0.5$. Recalling that $y=-4x$, we get the points $C(-0.5, 2)$; $D(0.5, -2)$.

72. **(a)** $4x-2(xy'+y)+2yy'=0$, so $y'=\dfrac{y-2x}{y-x}$. Tangent lines are horizontal where $y'=0$, which is where $y-2x=0$, in which case $y=2x$. Letting $y=2x$ in the equation of the ellipse, we obtain $2x^2-2x(2x)+(2x)^2-4=0$. Solving yields $x=\pm\sqrt{2}$. Recalling that $y=2x$, we get the points $A\left(-\sqrt{2}, -2\sqrt{2}\right)$; $B\left(\sqrt{2}, 2\sqrt{2}\right)$.

(b) Again, $y'=\dfrac{y-2x}{y-x}$. Tangent lines are vertical where y' is undefined, which is where $y-x=0$, in which case $y=x$. Letting $y=x$ in the equation of the ellipse, we obtain $2x^2-2x(x)+(x)^2-4=0$. Solving yields $x=\pm 2$. Recalling that $y=x$, we get the points $C(-2, -2)$; $D(2, 2)$.

73. **(a)** $2x-2(xy'+y)+2yy'-4=0$, so $y'=\dfrac{y-x+2}{y-x}$. The tangent line is vertical where y' is undefined, which is where $y-x=0$, in which case $y=x$. Letting $y=x$ in the equation of the parabola, we obtain $x^2-2x(x)+x^2-4x=8$. Solving yields $x=-2$. Recalling that $y=x$, we get the point $A(-2, -2)$.

(b) Again, $y'=\dfrac{y-x+2}{y-x}$. The tangent line is horizontal where $y'=0$, which is where $y-x+2=0$, in which case $y=x-2$. Letting $y=x-2$ in the equation of the parabola, we obtain $x^2-2x(x-2)+(x-2)^2-4x=8$. Solving yields $x=-1$. Recalling that $y=x-2$, we get the point $B(-1, -3)$.

74. Letting $x=0$ in the equation of the parabola, we get $y^2=8$, which has solutions $y=\pm 2\sqrt{2}$. Letting $y=0$ in the equation of the parabola, we get $x^2-4x=8$, which has solutions $x=2\pm 2\sqrt{3}$.
Implicitly differentiating, $2x-2(xy'+y)+2yy'-4=0$, so $y'=\dfrac{y-x+2}{y-x}$.
At y-intercept $\left(0, 2\sqrt{2}\right)$ the slope is $\dfrac{2\sqrt{2}-0+2}{2\sqrt{2}-0}=\dfrac{2+\sqrt{2}}{2}$.
At y-intercept $\left(0, -2\sqrt{2}\right)$ the slope is $\dfrac{-2\sqrt{2}-0+2}{-2\sqrt{2}-0}=\dfrac{2-\sqrt{2}}{2}$.
At x-intercept $\left(2+2\sqrt{3}, 0\right)$ the slope is $\dfrac{0-\left(2+2\sqrt{3}\right)+2}{0-\left(2+2\sqrt{3}\right)}=\dfrac{\sqrt{3}}{\sqrt{3}+1}$.
At x-intercept $\left(2-2\sqrt{3}, 0\right)$ the slope is $\dfrac{0-\left(2-2\sqrt{3}\right)+2}{0-\left(2-2\sqrt{3}\right)}=\dfrac{\sqrt{3}}{\sqrt{3}-1}$.

75. Every sinusoid with amplitude 3 and period π is the graph of some equation of the form $y = 3\sin(2x + C) + D$. The slope at any x is $\frac{dy}{dx} = 6\cos(2x + C)$. The maximum value of cosine is 1, so the maximum slope is 6.

76. Every sinusoid with amplitude A and period p is the graph of some equation of the form $y = A\sin\left(\frac{2\pi}{p}x + k\right) + D$. The slope at any x is $\frac{dy}{dx} = A \cdot \frac{2\pi}{p}\cos\left(\frac{2\pi}{p}x + k\right)$. The maximum value of cosine is 1, so the maximum slope is $\frac{2\pi A}{p}$.

77. Let $f(x) = \sin(x - \sin x)$. Then

$$f'(x) = \cos(x - \sin x)\frac{d}{dx}(x - \sin x)$$
$$= \cos(x - \sin x)(1 - \cos x).$$

This derivative is zero when $\cos(x - \sin x) = 0$ (which we need not solve) or when $\cos x = 1$, which occurs at $x = 2k\pi$ for integers k. For each of these values,

$$f(x) = f(2k\pi) = \sin(2k\pi - \sin 2k\pi)$$
$$= \sin(2k\pi - 0) = 0.$$

Thus, $f(x) = f'(x) = 0$ for $x = 2k\pi$, which means that the graph has a horizontal tangent at each of these values of x.

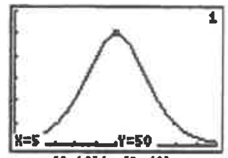
78. **(a)** $P(0) = \frac{200}{1 + e^5} \approx 1.339$, so initially one student was infected.

(b) $\lim\limits_{h\to\infty} P(t) = \lim\limits_{h\to\infty} \frac{200}{1 + e^{5-t}} = \frac{200}{1}$
$= 200$ students

(c) $P'(t) = \frac{d}{dt}200(1 + e^{5-t})^{-1}$

$$= -200(1 + e^{5-t})^{-2}(e^{5-t})(-1)$$
$$= \frac{200e^{5-t}}{(1 + e^{5-t})^2}$$

A graph of the derivative $y = P'(t)$ shows a maximum value at $t = 5$, at which point $P'(t) = 50$. The spread of the disease is greatest at $t = 5$, when the rate is 50 students/day.



[0, 10] by [0, 60]

The maximum rate occurs at $t = 5$, and this rate is

$$P'(5) = \frac{200e^0}{(1 + e^0)^2} = \frac{200}{2^2} = 50 \text{ students per day.}$$

79. Differentiating implicitly,

$2x + 2(y + xy') + 4yy' = 0$, so $y' = -\frac{x + y}{x + 2y}$.

(a) At (1, 1) we get $y' = \frac{dy}{dx} = -\frac{2}{3}$.

(b)
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$
$$= \frac{d}{dx}\left(-\frac{x + y}{x + 2y}\right)$$
$$= -\frac{(1 + y')(x + 2y) - (1 + 2y')(x + y)}{(x + 2y)^2}$$

At (1, 1) and recalling that $y' = \frac{dy}{dx} = -\frac{2}{3}$, we get

$$\frac{d^2y}{dx^2} = -\frac{\left(1 - \frac{2}{3}\right)(1 + 2) - \left(1 - \frac{4}{3}\right)(1 + 1)}{(1 + 2)^2}$$
$$= -\frac{5}{27}.$$

80. Use implicit differentiation.

$$x^2 - y^2 = 1$$
$$\frac{d}{dx}(x^2) - \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$
$$2x - 2yy' = 0$$
$$y' = \frac{2x}{2y} = \frac{x}{y}$$
$$y'' = \frac{d}{dx}\frac{x}{y} = \frac{(y)(1) - (x)(y')}{y^2}$$
$$= \frac{y - x\left(\frac{x}{y}\right)}{y^2} = \frac{y^2 - x^2}{y^3} = -\frac{1}{y^3}$$

(because the given equation is $x^2 - y^2 = 1$)

At $\left(2, \sqrt{3}\right)$, $\frac{d^2y}{dx^2} = -\frac{1}{y^3} = -\frac{1}{(\sqrt{3})^3} = -\frac{1}{3\sqrt{3}}$.

81. (a) $g'(x) = k \cdot e^{kx} + f'(x)$, so $g'(0) = k + 3$.

$g''(x) = k^2 \cdot e^{kx} + f''(x)$, so

$g''(0) = k^2 - 1$.

(b) $h'(x) = -b\sin(bx)f(x) + f'(x)\cos(bx)$, so $h'(0) = -b \cdot \sin(0) + 3 \cdot \cos(0) = 3$. Note that $h(0) = \cos(0) \cdot f(0) = 1 \cdot 2 = 2$, so the tangent line has equation $y - 2 = 3(x - 0)$ or $y = 2 + 3x$.

82. (a)
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(e^x + e^{-x})\right) = \frac{1}{2}(e^x + e^{-x}(-1)) = \frac{e^x - e^{-x}}{2}$$

(b)
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{2}(e^x - e^{-x})\right) = \frac{1}{2}(e^x - e^{-x}(-1)) = \frac{e^x + e^{-x}}{2}$$

(c) $y(1) = \dfrac{e + e^{-1}}{2} \approx 1.543$;

$\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{e - e^{-1}}{2} \approx 1.175$;

$y = 1.543 + 1.175(x - 1)$

(d) $m = -\dfrac{1}{dy/dx} = -\dfrac{2}{e - e^{-1}} \approx -\dfrac{1}{1.175} \approx -0.851$

$y = 1.543 - 0.851(x - 1)$

(e) $\dfrac{e^x - e^{-x}}{2} = 0$ $\left(\text{set } \dfrac{dy}{dx} \text{ equal to } 0\right)$

$e^x - e^{-x} = 0$ (multiply both sides by 2)

$e^x = e^{-x}$

$e^{2x} = e^0$ (multiply both sides by e^x)

$\ln\left(e^{2x}\right) = \ln e^0$

$2x = 0$

$x = 0$

The tangent line is horizontal at $x = 0$.

83. (a) $1 - x^2 > 0 \to x^2 < 1 \to \sqrt{x^2} < \sqrt{1} \to |x| < 1 \to -1 < x < 1$

Domain of $f = (-1, 1)$

(b) $f'(x) = \dfrac{1}{1 - x^2}(-2x) = -\dfrac{2x}{1 - x^2}$

(c) Domain of $f' = \{x \mid x^2 \neq 1$ and $x \in$ domain of $f\}$

Domain of $f' = (-1, 1)$

(d)
$$f''(x) = -\frac{(1 - x^2)(2) - (-2x)(2x)}{(1 - x^2)^2} = -\frac{2 - 2x^2 + 4x^2}{(1 - x^2)^2} = -\frac{2 + 2x^2}{(1 - x^2)^2} = -\frac{2(x^2 + 1)}{(x^2 - 1)^2} < 0 \text{ for } x \neq \pm 1$$

(The numerator and denominator are clearly both positive.) Therefore, $f''(x) < 0$ for all $x \in (-1, 1)$.